# FEC Rubric

**Engineering Notebook**

| | | | |
|---|---|---|---|
| 1. Explanation of Motor Curves | Fair -1 | Good -3 | Excellent -5 |
| 2. Motor Curve Construction | Fair -1 | Good -3 | Excellent -5 |
| 3. Torque vs. Current Discussion | Fair -1 | Good -3 | Excellent -5 |
| 4. Lift Time Calculation | Fair -1 | Good -3 | Excellent -5 |
| 5. Thermal Mass & Torque Discussion | Fair -1 | Good -3 | Excellent -5 |
| 6. Optional Gearing Arrangements | Fair -1 | Good -3 | Excellent -5 |
| 7. Motor Efficiency calculations | Fair -1 | Good -3 | Excellent -5 |
| 8. Choosing Velocity vs. Time Discussion | Fair -1 | Good -3 | Excellent -5 |
| 9. Calculating actual time to lift the mass | Fair -1 | Good -3 | Excellent -5 |
| 10. Calculation of possible repetitions | Fair -1 | Good -3 | Excellent -5 |
| 11. Deriving Torque vs. Angular Velocity | Fair -1 | Good -3 | Excellent -5 |
| 12. Deriving Motor Power Curve | Fair -1 | Good -3 | Excellent -5 |
| 13. Following all Engineering Notebook instructions | Fair -1 | Good -3 | Excellent -5 |

**Presentation:**

| | | | |
|---|---|---|---|
| Theory: Analysis and Calculations | Fair -1 | Good -3 | Excellent -5 |
| Creativity | Fair -1 | Good -3 | Excellent -5 |
| Coopetition | Fair -1 | Good -3 | Excellent -5 |
| Presentation Clarity | Fair -1 | Good -3 | Excellent -5 |

**Prototype Demonstration**

| | | | |
|---|---|---|---|
| Summary of Theoretical Operation, including numbers | Fair -1 | Good -3 | Excellent -5 |
| Demonstration of Actual Operation, including numbers | Fair -1 | Good -3 | Excellent -5 |
| Explanation of the Difference between Theory and Actual numbers | Fair -1 | Good -3 | Excellent -5 |
| | | **Total** | **100pts** |

# Skunkworks Robotics

# Team 1983

**1983**

## Skunkworks Robotics

### Engineering Challenge Student Participants

| | | | |
|---|---|---|---|
| Aiden Fellers | 11th | 1) | Explanation of Motor Curves 2)Motor Curve Construction |
| Jake Hecla | 11th | 3) | Torque vs. Current Discussion |
| | | 11) | Deriving Torque vs. Angular Velocity |
| | | 12) | Deriving Motor Power Curve |
| Reilly Mull | 11th | 4) | Lift Time Calculation |
| Liam Burke | 11th | 5) | Thermal Mass & Torque Discussion |
| Barrett Estep | 11th | 6) | Optional Gearing Arrangements |
| Quinn S | 9th | 7) | Motor Efficiency calculations |
| Davis Yoshida | 12th | 8) | Choosing Velocity vs. Time Discussion |
| | | 9) | Calculating actual time to lift the mass |
| Joshua Stuech | 9th | 10) | Calculation of possible repetitions |
| Hunter McSwain | 11th | | Prototype Demonstration |
| Camila Palacio | 10th | 13) | Following all Engineering Notebook instructions |

### Engineering Challenge Mentor Participants

Ken Zaballos

Charlie Perkins

Edward Yoshida

Jim DeSilva

Tim Davis

### Contact Phone Number and Email Address

Ken.zaballos@gmail.com, (206) 409-2165

### School Name

Aviation High School

**Problem Statement:**

In the First Robotics Competition, electric motors are used in a variety of ways to do physical work. Understanding the capabilities and limitations of the motors in your arsenal will allow you to better choose and design a mechanism to accomplish a task in an optimal way. Consider a problem where you need to lift a 5-kg mass from the floor to a height of 2 meters. Each motor in your kit of parts can accomplish this task, given appropriate gearing reductions. That being said, the amount of time it would take to accomplish the task would vary significantly depending on the properties of the motor.

Consider the problem stated previously, where a 5-kg mass is lifted to a height of 2 meters above the ground. Your challenge is to choose a motor from the 2011 FRC kit of parts and design an associated mechanism to accomplish this task where the motor is operating at or near its maximum power output. You will be judged on your analysis of the problem and your ability to convey to a panel of judges the science and reasoning behind your decisions.

Consider quantities such as torque, work, power, acceleration, current, efficiency, friction and thermal mass in your analysis. Realize that this is not a comprehensive list, but it should serve as a starting point.

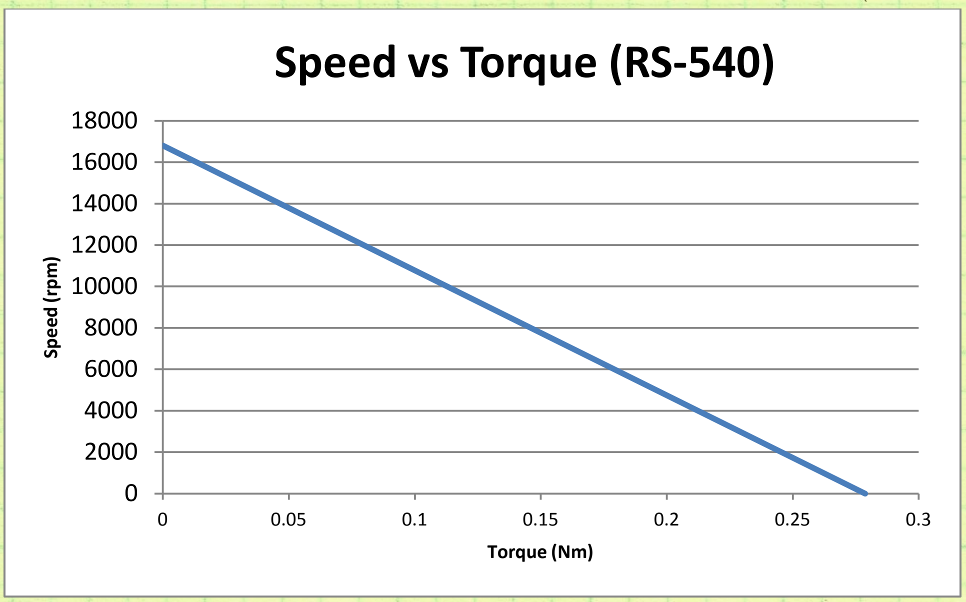
# #1

FIRSTWA ENGINEERING CHALLENGE 2011

1. Explain what the Motor Curves show and how we use them to predict motor function by performing the following steps:

a. Draw a set of motor curves for a motor in your teams FIRST parts kit

**The motor that our team chose for this problem was the Banebots RS-540.**




The "Speed vs. Torque" curve above shows the linear relationship between motor speed and torque. The maximum speed of 16800 rpm occurs when no-load is applied to the motor shaft. The minimum speed of 0 rpm occurs with a stall torque of 0.28 Nm. These are specifications provided by Banebots LLC (http://banebots.com/p/M2-RS540-120). Between these two points there is a linear relationship because speed starts from no-load torque and decreases to stall at a proportional rate. Speed is the motor's revolutions per minute variable and torque is the twisting force the motor experiences causing its shaft to turn. The "Speed vs. Torque" curve shows the speed in rpm that the motor will produce for any given torque

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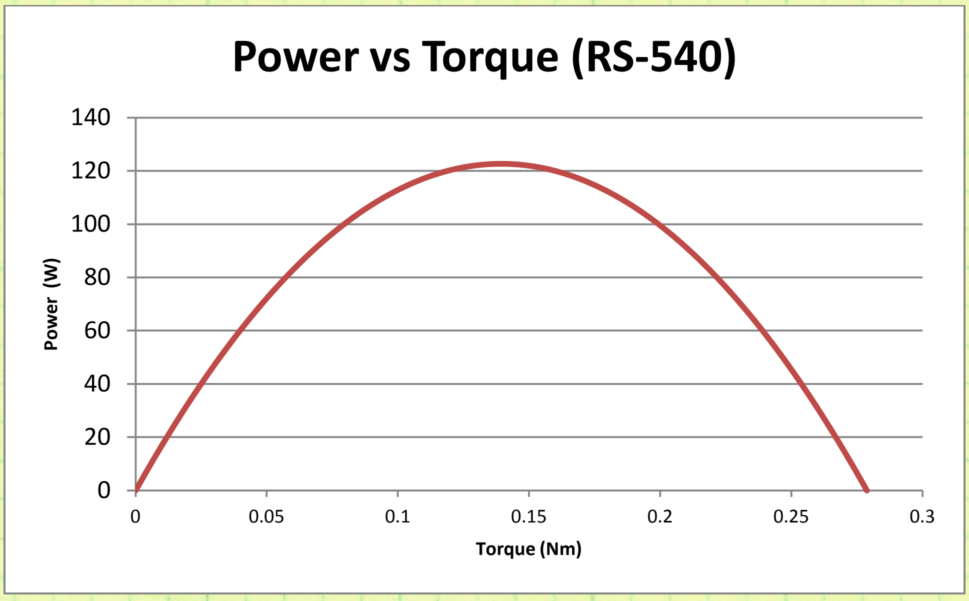
1. Continued




The “Power vs Torque” curve shows a parabolic relationship between motor power and torque based upon the following equation.

$$P = -(\omega_{free} / \tau_{stall}) \cdot \tau^2 + \tau \cdot \omega_{free}$$

Where $\omega_{free}$ is the free speed of the motor with no load

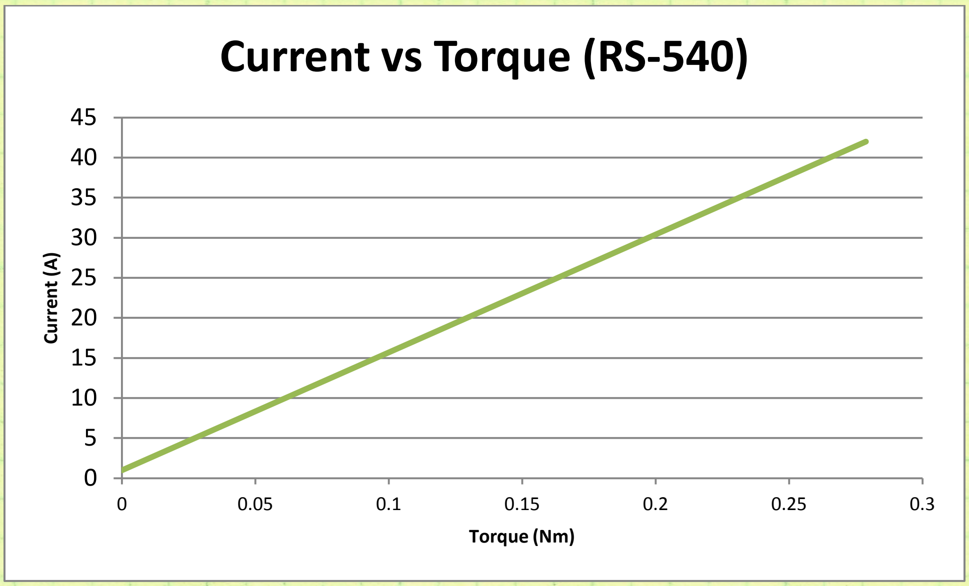
$\tau_{stall}$ is the stall torque of the motor, and

$\tau$ is the torque for which power is being calculated.

The maximum power of 122.6 W occurs at the mid-point between no-load torque and stall torque. The minimum power of 0 W occurs when torque is either at no-load or stall. Between these two points it is a parabola relationship because solving for power would create an inverse of torque$^2$.

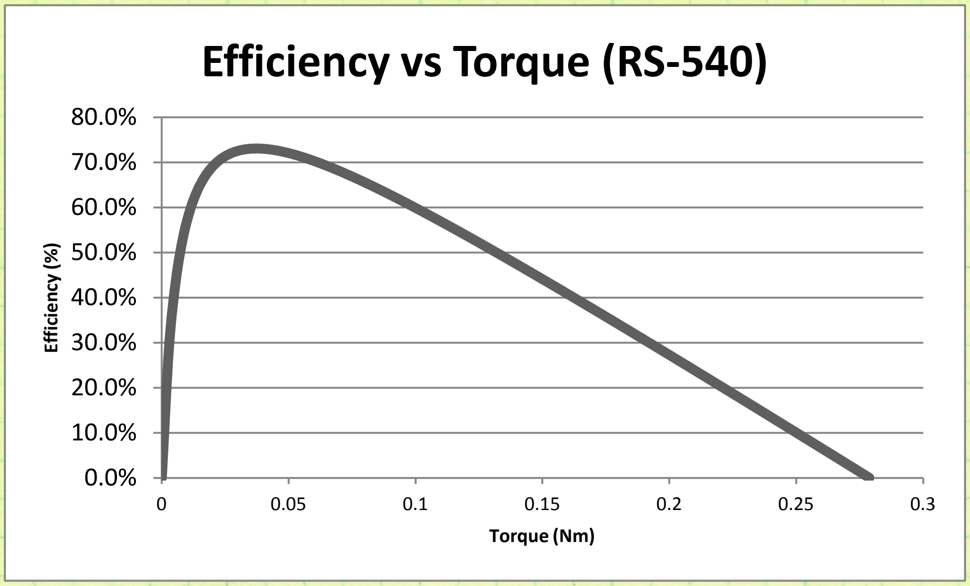
Power is the rate at which energy is delivered and torque is the twisting force the motor experiences causing its shaft to turn. The “Power vs. Torque” curve shows the power the motor will produce for any given torque.

1. Continued




The “Current vs Torque” curve shows a linear relationship between motor current and torque. The maximum current of 42 amps occurs at stall torque. The minimum current of 1 amp occurs because of the losses inside the motor. Examples of losses would be electrical resistance of the motor windings and friction. There will always need to be some current supplied to the motor to keep the motor rotating. The relationship between current and torque is linear because torque increases at a proportional rate as current is applied to the motor. Current is the flow of electrons inside the motor and torque is the twisting force the motor experiences causing its shaft to turn. The “Current vs Torque” curve shows the more current that goes through the motor the more torque that is created.

1. Continued




"Efficiency vs. Torque" curve shows a relationship between motor efficiency and torque. The maximum efficiency of 73.0% occurs when torque is at 0.039032 Nm. The minimum efficiency of 0% occurs when torque are either at no-load or stall. If the efficiency is at 0% because torque is at stall torque then this will cause heat in the motor. Efficiency is the ratio of energy delivered by a motor to the energy supplied to it during a fixed period or cycle and torque is the twisting force the motor experiences causing its shaft to turn. "Efficiency vs. Torque" curve shows the efficiency of the motor and how much torque can be applied before heat is made.

1. Continued

b. Explain each motor **variable** that **is** plotted on a set of motor curves along with the variable's physical meaning.

**Efficiency:** The ratio of energy delivered by a motor to the energy supplied to it during a fixed period or cycle.

$$e = \frac{P_{out}}{P_{in}}$$

**Torque:** The twisting force the motor experiences causing its shaft to turn. It has units of Nm and is the independent variable.

**Power:** The rate at which energy is delivered and has units in watts. It is an inverted parabola. Its peak occurs at ½ the stall torque.

$$P = -(\varpi_{free} / \tau_{stall}) \cdot \tau^2 + \tau \cdot \omega_{free}$$

**Current:** The flow of electrons in the motor. It has a linear relationship with torque.

$$i = \frac{i_{free} - i_{stall}}{0 - \tau_{stall}} * \tau + i_{free}$$

**Speed:** The motor's revolutions per minute variable. It has a linear relationship with torque.

$$\omega = \frac{(\omega_{free})}{-\tau_{stall}} * \tau + \omega_{free}$$

1. Continued

c. Explain how you use motor curves to predict motor performance.

The motor curves can be used to predict motor performance in a variety of ways. In this application, we were given a requirement that the motor operate at, or near, peak power when lifting the mass. This dictated a particular amount of torque available for each motor. At this torque, by definition ½ stall torque, the speed of the motor, the current it draws, and the efficiency can be determined.

Current draw is very important because the control system has circuit breakers in the motor circuit that will trip if current is greater than 40 amps for more than a few seconds. One or our team design guidelines for this task was that the current draw at peak power could not exceed 40 amps.

The efficiency of the motor at the required torque indicates how much electrical energy is wasted. As long as the required torque is to the left of peak power, the wasted energy should be manageable. If the task is highly repetitive, or continuous, selecting a torque associated with peak efficiency might be the better choice since the least amount of electrical energy will be wasted.

Another way to approach use of these curves is to have a desired motor speed to perform a task. Given a speed, there are two possible power solutions. The one to the left of peak power should always be selected since the once to the right operates at a very low efficiency and risks damage to the motor and poor use of battery capacity. The torque associated with the selected power must be sufficient to handle the load. Again, current must be below 40 amps.

All of these analyses require proper consideration of the mechanical efficiency of the drive train to ensure the actual performance of the device closely matches the prediction.

# #2

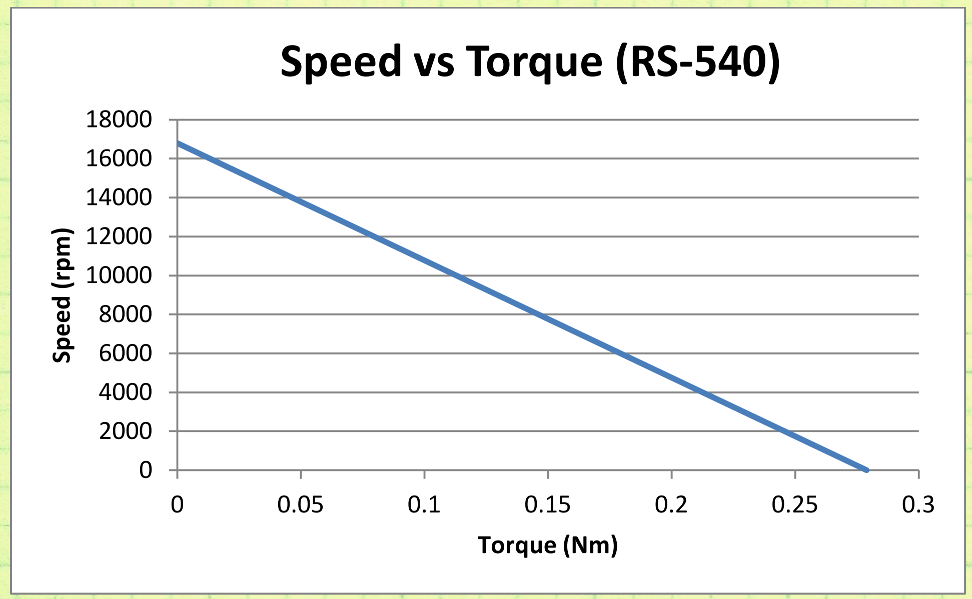
2. Using the data sheet for a motor in your teams FIRST kit of parts, explain how you use data from that sheet to construct the Torque vs. Speed Curve, the Current vs. Torque Curve, and the Power Curve. Use graphs along with words with each of the following explanations:

a. Torque vs. Speed Curve

To create the Torque vs. Speed curve you need the motor stall torque where the speed is equal to zero and the motor free speed where the torque is equal to zero. For the Banebots RS-540 motor, the stall torque is specified as 278.8 mN-m and the free speed is listed as 16,800 rpm. It is a straight line relationship between the two points. To get speed for a given torque we use this equation.

$$\omega = \frac{(\omega_{free})}{-\tau_{stall}} * \tau + \omega_{free}$$

Torque will always be on the x-axis and speed is placed on the y-axis.



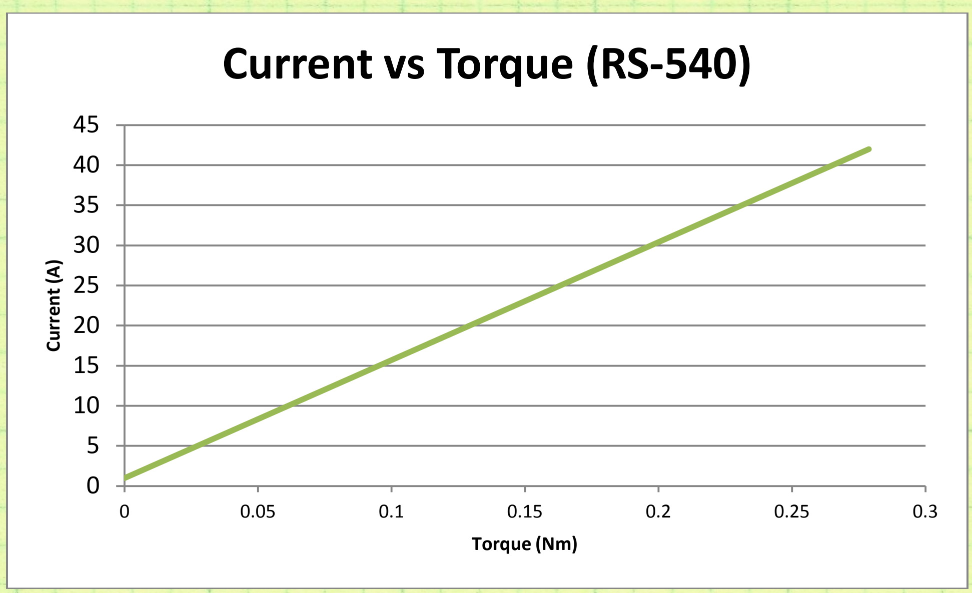
2. Continued

b. Current vs. Torque Curve

To get the Current vs. Torque curve we need the motor stall torque where current is equal to its stall current and the motor no-load torque where current will be equal to its minimum value. For the Banebots RS-540 motor, stall torque is listed as 278.8 mN-m and stall current is specified as 42 amps. It is a straight line relationship between the two points. To get current required for a given torque value we use this equation.

$$i = \frac{i_{free} - i_{stall}}{0 - \tau_{stall}} * \tau + i_{free}$$

Torque will always be on the x-axis and current is placed on the y-axis.



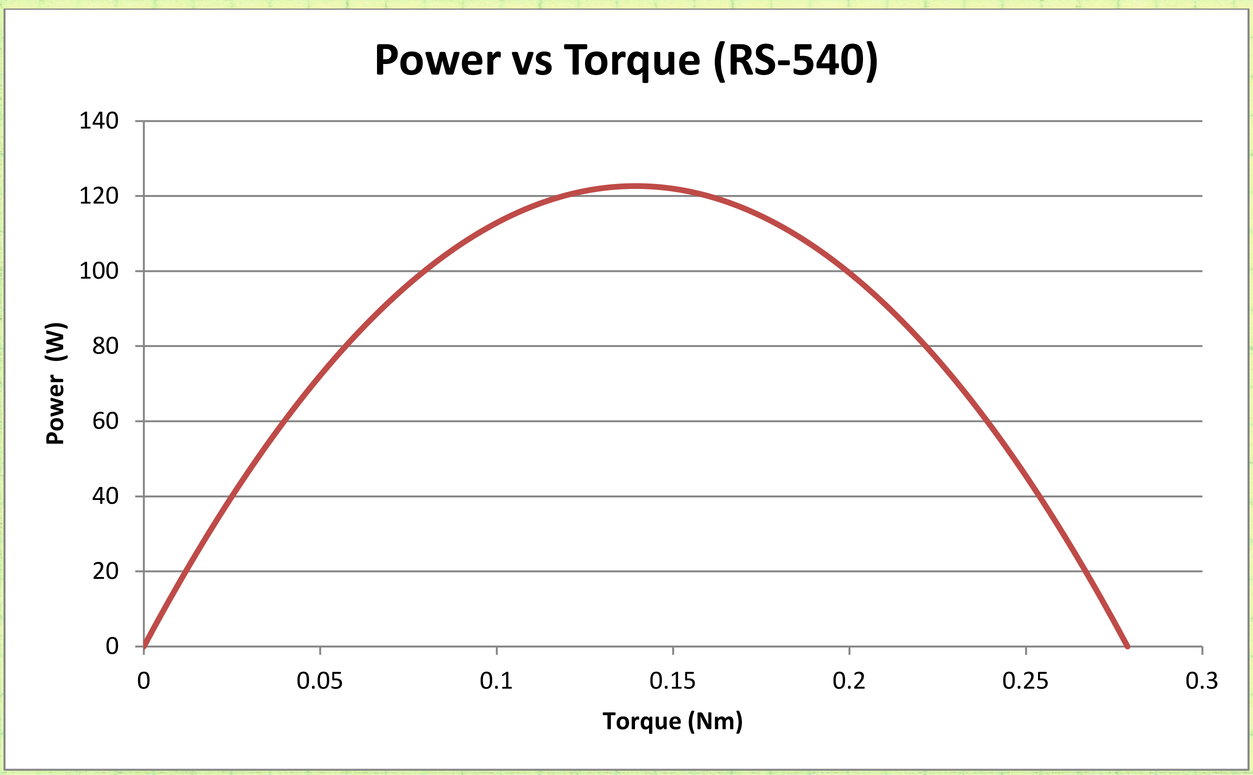
2. Continued

c. Power Curve

Power output of the motor is equal to the speed multiplied by torque. To get the Power vs. Torque curve we need motor torque and motor no-load speed to calculate power which is speed multiplied by torque. It is a parabolic relationship between these two variables. To calculate power we use this equation.

$$P = -(\varpi_{free} / \tau_{stall}) \cdot \tau^2 + \tau \cdot \omega_{free}$$

Torque will always be on the x-axis and power is placed on the y-axis.



#3

3. Discuss Torque vs. Current relationship for the motor

<u>Theory of Operation</u>

In a permanent magnet DC motor, the motor's torque is based on the strength of the force produced by the interaction of the current flowing through the windings and the EM field of the can magnets. DC electric motors function by passing current through wire coils attached to a moveable armature. The current flowing through the wires in the magnetic field creates a force called a Lorenz Force that is the cross product of the magnetic field and the direction of the current flow. If the force applied to the armature is significant enough to overcome the static friction in the motor, then the coil moves (and the armature along with it). This effect is turned into useable motion by the action of the commutator, which re-routes current to the next stack of windings (pole) to keep a consistent torque as the motor rotates. As the current increases, so does the force applied to the armature. This can be calculated (in its simplest form, assuming a single coil (one pole) in a single magnetic field without efficiency losses) using the equation:

$$\tau = (W)(I)(L)(B)\sin(\theta) \qquad \text{Equation 1}$$

Note: The direction of the force is the cross product of the magnetic field (B) and the current flowing through the wire, the magnitude of the value changes based on the coil's angle to the field, necessitating the use of the $sin\theta$ term.

The coil characteristics can be expressed as:

$$\mu = (I)(L)(W) \qquad \text{Equation 2}$$

Substitution to get:

$$\tau = \mu B sin(\theta) \qquad \text{Equation 3}$$

Units: (I) Current is in amps, (W/L) length is in meters, (B) magnetic field is in teslas and sine is in radians.

3. Continued

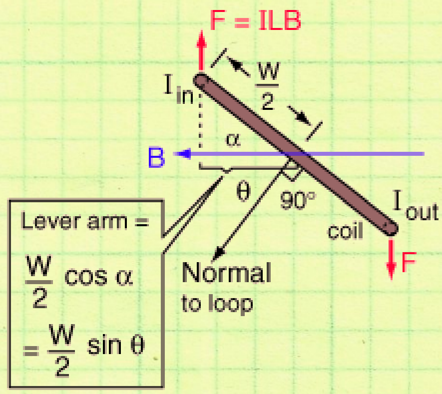



Photo courtesy of hyperphysics

Limitations

The torque delivered by the motor does not stay at stall torque all across the whole operating speed spectrum. As the motor spools up, the motor's available torque declines in a linear fashion. This is due to the influence of "back EMF" induced by the motion of the wire loops through the magnetic field. The opposition to current flow in the motor (and therefore the loss of torque) is caused by the wire conductors moving through the magnetic field, causing induction that is opposite to the input current. Just as a motor converts electrical energy to mechanical motion, the motor converts its motion back to electrical energy (hence the term "generator effect"), albeit in the opposite direction. As speed increases, so does the back EMF generated. As the motor spins faster, the net current (and consequently the "$\mu$" term) grow smaller. Eventually, the input voltage (minus resistance and inefficiencies) and back EMF become equal, and the net torque on the armature reaches zero. At this point, the motor has reached max free speed. In a standard DC motor, this is accounted for as the constant Kt (sometimes expressed as Kg) that determines torque loss per unit of current applied. For our purposes, this relationship can be considered perfectly linear. In the case of the RS 540, the Kt value is roughly 6.6 mN/amp, which intercepts the y axis at 1,770 radians/sec (16,800 rpm).

3. Continued

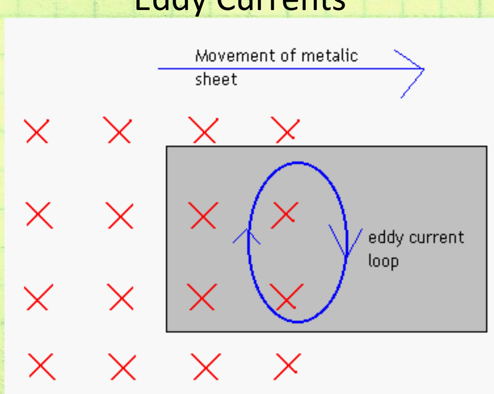
Efficiency and Losses

In addition to back EMF, the mechanical friction and electrical resistance inherent in the motor limit available torque and maximum rotation speed. This is clearly demonstrated by examining the no load current, as almost all of that is used in overcoming mechanical losses.

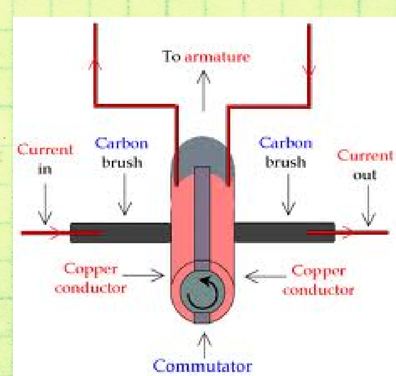
Most of the losses in any DC motor come from friction between the bearings/bushings supporting the armature shaft. In addition, there are smaller losses in the commutator, resistance and in eddy currents. The most significant of these, commutator losses, occur both through resistance and sparking caused by the coils' inductance. The commutator is a segmented barrel shaped conductor with slots running lengthwise that redirects current. Two brushes (usually contoured blocks of a soft copper or silver alloy) rest against the barrel at $90^o$ to the axis and pass current to two segments of the commutator at a time. When the motor spins, the commutator brushes slide over each section and provide current to the coils that keep the motor advancing. The losses in this system occur when energy stored in the EM field of the coil is dumped as the commutator slides to the next section. When this occurs, a significant arc can be seen from the brush to the last segment, emitting energy in the form of photons and heat.

Another loss that occurs in the system is inductive heating triggered by eddy currents. Though most modern motor systems use laminated cores to minimize this, some older or less sophisticated iron core motors have significant losses associated with eddy currents. The conductive metal moving through the magnetic field induces a current within itself that is not uniform and that stores some energy in the field it creates. Losses arise from both the inductance and through heating the iron core (from the current passing through it).

Eddy Currents




Commutator diagram



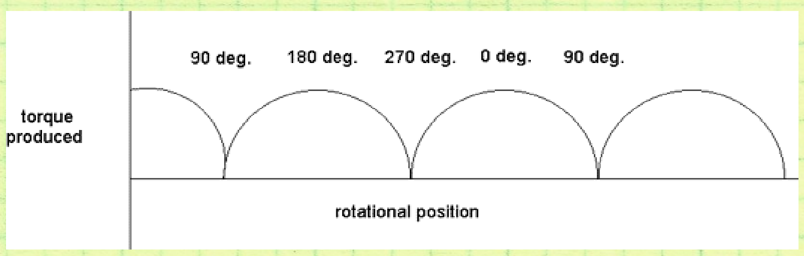

Photos courtesy of University of Iowa website

3. Continued

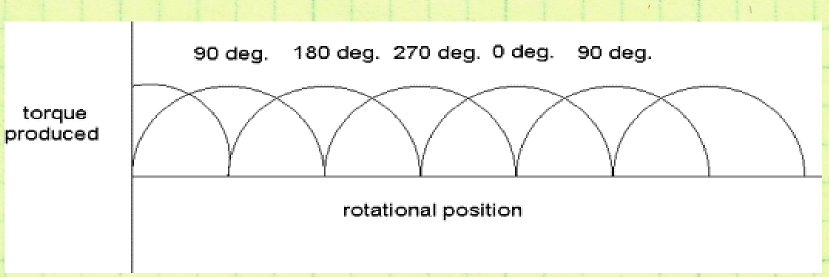
DC Motor Design

As expressed in equation 3, the torque available in a permanent magnet DC motor is dependent on the powered coil's angle to the field. This varies considerably based on the position of the motor through its rotation. Looking at this variation on a graph yields a cycloid-like pattern known as a "torque ripple". If only one coil is present, the torque on the goes to zero every $90^{o}$ of rotation (effectively not making it a motor), but trends closer and closer to a line as more poles are added. Depending on the task for which you are employing the motor, this can greatly modify the overall performance and efficiency of the system.

Three pole motor




Multipole motor



# #4

4. Explain and show calculations for minimum time to lift 5kg two meters. Assume no friction.

The next step in our process was to determine the amount of time that it would take us to lift a 5kg weight 2 meters using the Banebots RS-540 motor. In order to do this, we started with the definition of power: the amount of work required for an action over a period of time. From this, we were able to derive this simple equation:

$$\text{Time}=\frac{\text{Work}}{\text{Power}}$$

We had already found the amount of power that we were going to be using (122.8 Watts), so we just needed to find the work. Work is found by multiplying the force applied to an object as it is moved a distance.

$$\text{Work}=\text{Force} \times \text{Distance}$$

We realized that we also needed to find the amount of force required to move the object. This is found by multiplying the mass of the object (in kilograms) by the acceleration of gravity (in meters per second per second) acting upon it. We are assuming that the object is moving at a constant velocity during the entire motion.

$$\text{Force}=\text{Mass} \times \text{Acceleration}=5\text{kg} \times 9.8\text{m/s}^2 = 49 \text{ Newtons}$$

We then plugged 49 Newtons of force into the equation for work:

$$\text{Work}=\text{Force} \times \text{Distance}=49 \text{ Newtons} \times 2\text{m}=98 \text{ Joules}$$

Finally, we plugged 98 Joules of work into our original equation to get the number of seconds necessary to raise the 5kg weight 2 meters into the air:

$$\text{Time}=\frac{\text{Work}}{\text{Power}} = \frac{98 \text{ Joules}}{122.8 \text{ Watts}} = 0.8 \text{ Seconds}$$

From these calculations, we found that it would take 0.8 seconds to raise the weight if you considered an ideal system without mechanical losses of acceleration from when it starts at a velocity of zero, goes to a steady velocity, and returns to zero velocity.

#5

5. Evaluate and discuss a motor's thermal mass and current required for various torques.

To understand Thermal Mass and Torque and how they are connected, it is first necessary to understand the components of the motor that are concerned. The coil of wire inside a motor consists of many loops (one wire), and it is attached to the rotor of the motor and rotates. As it does so, the magnitude and direction forces on the wires remain approximately constant. The force generated on each effective segment of the loop is called a Lorentz force, which contributes to the torque delivered by the motor. This torque is directly proportional to the number of loops in the coil and/or the current flow. With more loops in the coil, or the more current flowing through the coil, more force is applied to the rotor, which in turn generates more torque.

The thermal failure of a DC motor is when the wire insulation melts, or burns away. When this happens, the coil shorts out, damaging the motor. In FIRST Robotics, this is called "Letting out the magic smoke"

Thermal energy is being delivered to the coil at a rate of: $Pthermal = i^2 * R$ (Joules/second)

The change in temperature of the wire is described as: $Q = M*C*deltaT$ (Joules)

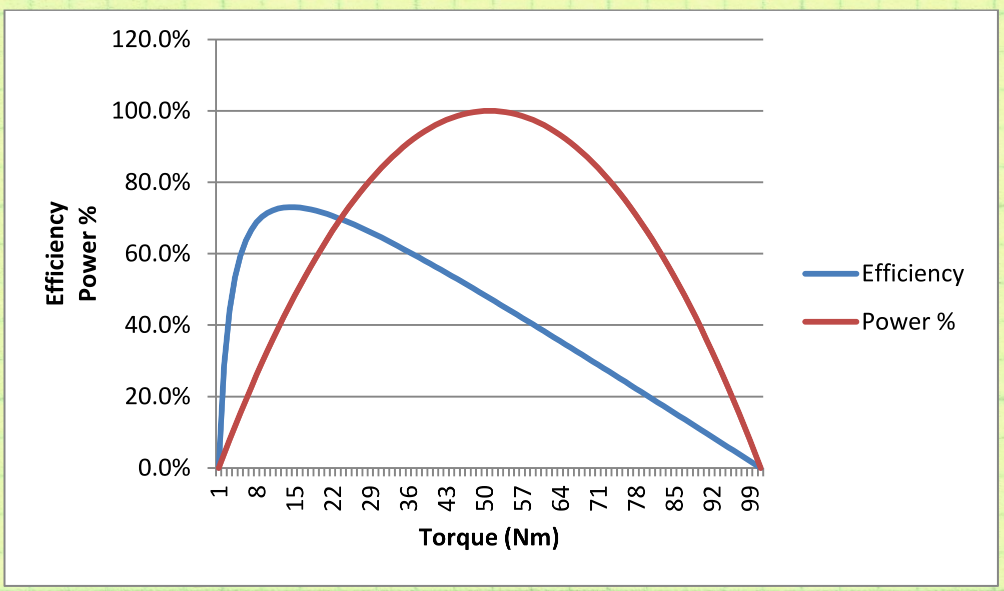
In this equation, m is the thermal mass of the motor, c is the specific heat capacity of the motor (mostly copper), q is the thermal energy put into the motor and deltaT is the change in temperature.

The thermal mass of the motor primarily includes all of the components of the armature of the motor (coil, core, shaft, commutator, etc.) The energy is generated within the wire, however, so it heats up fastest and is the closest to the component most venerable to heat, the insulation. The wire insulation will melt when the wire reaches a particular temperature.

5. Continued

Heat energy is being generated with the square of the current, so a small mass motor will heat up more quickly at a given current than a large mass motor. As the number of wire loops increases or the diameter of the wire increases, the greater the thermal mass of the motor. This generally means a bigger and more powerful motor, and because of all this they heat up more slowly at a given current level.

The Banebots RS540 is a small motor and is relatively easy to overheat if run improperly. Given the parabolic nature of the power curve of a motor in general, any given shaft speed can be achieved with two different torque values. Torque values to the left of the peak power on the curve generate heat that can be dissipated by airflow through the motor. Torques to the right of peak power cannot dissipate heat effectively as the speed of the motor slows down. The amount of energy to dissipate increase very quickly, as well. The torque to the left of the peak will operate all day long, while the torque to the right of the peak will not perform for long durations and may melt the wire insulation.



# #6

6. Explain your evaluations of various gearing arrangements, their losses, and why you are choosing the one you have.

We limited our design to address the following:

- Availability of components in our shop
- Level of complication and need for fabricating parts
- Speed of assembly
- Rigidity of the structure
- Current draw had to be <40A

On hand we had these gearboxes:

| Model | Reduction | Efficiency |
|---|---|---|
| CIMple Box | 4.67:1 | 98% |
| Tough Box | 12.75:1 | 94% |
| AM Planetary | 3.67:1 | 86% |
| CIM-u-lator | 2.7:1 | 98% |
| P60s | 16:1, 44:1, 128:1, 256:1 | 60%-90% |

According to our calculations and efficiency assumptions, the correct ratio was calculated to be 17.15:1, which could be achieved with several combinations of gearboxes and chain drives.   The calculation for this is documented below. The final design of our system uses a CIMple box (3.67:1) connected to an AM planetary (4.67:1) to achieve the desired gear ratio. Fortunately, we had a drum left over from the 2008 season with a diameter of 3.56 in. This drum was ideal because no modification was required to connect to the CIMple Box output shaft. One feature of the CIMple box is that the output shaft is cantilevered, making the box easier to incorporate into the system. Within the CIMple box is a double bearing configuration that gives the shaft adequate support for this design.

6. Continued

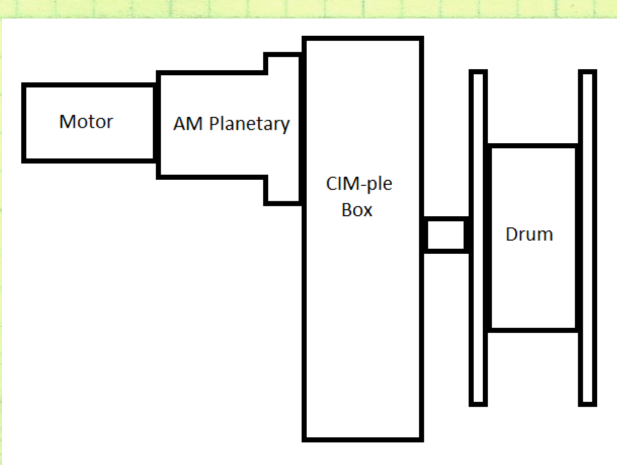
In addition, the CIMple box has an encoder. The encoder is an electronic device connected to the end of the drive shaft that generates a stream of data the control system can read to determine the speed and position of the mass. This enables the controller to know when to stop the mass (when it reaches two meters).

The runner up to the CIMple box was the Tough Box. This gearbox had both the encoder and double bearing design. However, it differed from the CIMple box in that its gear ratio (12.75:1), when connected to the AM Planetary, was not adequate given the drum we had decided to use. To modify the gear ratio and make it viable would require an additional chain drive. We wanted to avoid this because it required more engineering, more build time, and has somewhat decreased mechanical efficiency.

A feature of the AM Planetary is its compatibility with the RS-540 motor. The AM Planetary is designed for compatibility with the Fischer-Price motor, which shares the same mounting configuration as the RS-540.

**Losses in a drivetrain**

In any gearbox, chain drive or pulley system, some energy is lost to friction. This can come in a variety of forms, including gear mesh interaction, bearing friction and energy lost churning grease in the gears. Some gearbox specifications represent this by an efficiency rating (usually given in percent). In the AM planetary system, this rating was assumed to be 96% and was not quoted by the manufacturer. The CIMple box was assumed to be slightly higher at 98% because of its relatively simple design. Depending on the individual gearbox and the tolerances to which it is made, this can vary greatly.




Picture and diagram of the motor-gearbox-drum assembly:

6. Continued

Calculations regarding the gear ratio:

Diameter of the drum:

$$d_{drum} = 3.5625\ in$$

$$d_{drum} = 3.5625\ in * 0.0254 \frac{m}{in} = 0.0905\ m$$

Force:

$$F = 5 * 9.81\ \ kg \frac{m}{s^2} = 49.05\ N$$

Torque of the drum:

$$\tau_{drum} = F * (\frac{d_{drum}}{2})$$

$$\tau_{drum} = 49.05\ N * \left(\frac{.0905\ m}{2}\right) = 2.22\ Nm$$

Torque of motor at peak power:

$$\tau_{motor} = .139Nm$$

Ideal Reduction:
This assumes no losses.

$$R = \frac{\tau_{drum}}{\tau_{motor}}$$

$$R = \frac{2.22\ Nm}{.139\ Nm} = 15.97 \text{ Reduction for ideal gearbox}$$

6. Continued

Efficiencies:
CIMple box: 98%
AM Planetary: 96%
Drum: 99%

Total mechanical efficiency: 93.1%

Only 93.1% of the mechanical power delivered by the motor is available at the output shaft. 6.9% of the power is wasted due to friction in the gearbox. The reduction must be increased by the inverse of the efficiency so that the power delivered by the motor equals the desired amount.

Reduction with Losses:

$$R = \frac{\tau_{drum}}{\tau_{motor} * efficiency}$$

$$R = \frac{2.22Nm}{.139Nm * 93.1\%} = 17.15$$ Required reduction

Total Reduction of actual gearboxes:

$$R_{Total=} R_{AM\ planetary} * R_{CIMple\ box}$$

$$R_{Total=} \left(\frac{3.67}{1}\right) * \left(\frac{4.67}{1}\right)$$

$$R_{Total=} 17.14$$ Reduction of actual gearbox

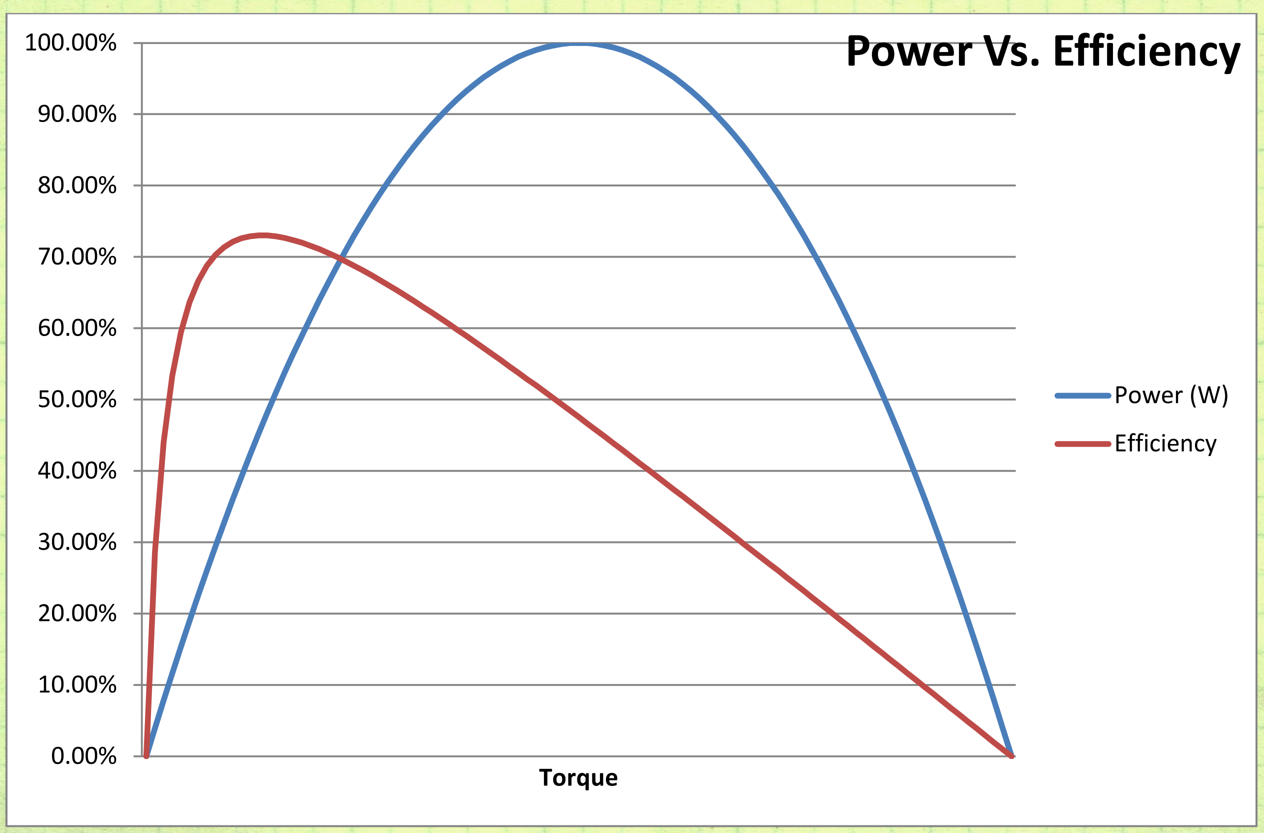
# #7

7. Show a graph of your motor's efficiency vs. torque and discuss at what range of torques and efficiencies you will operate.

When the motor is turned on, the torque is at stall. This point is on the far right of the graph. As the motor accelerates, the power increases until it reaches peak power. All of the lifting with the exception of the time spent accelerating will occur at peak power.

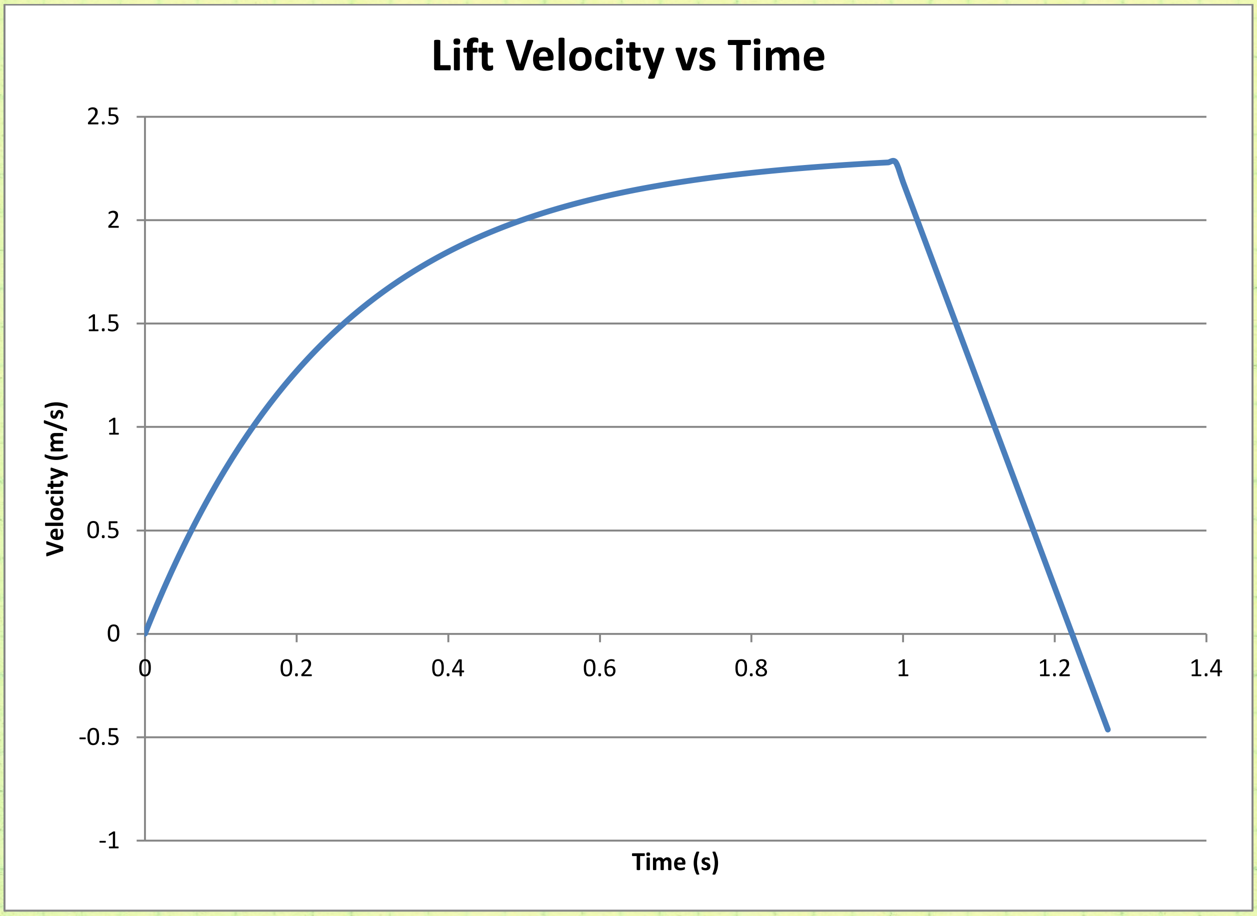
Once the weight reaches the top, the motor will be operating at stall torque again. At stall torque, the efficiency is zero. Therefore, we cannot operate there for long or else the insulation in the motor will melt.

The torque range the system operates in is from stall toque to one-half stall toque. The power range is from peak power and to the right. At peak power, the Banebots RS 540 motor uses 21.09 Amps and returns 122 Watts of power. It is only 47% efficient at this point. This means that 53% of the power input to the motor is being converted into thermal energy and performs no useful work. Since the efficiency at peak power is only 47%, the range of efficiency is from 47% at peak power and to zero. The motor never operates at peak efficiency.



#8

8. a. Plot the velocity vs. time graph that you will use for lifting the 5kg two meters.



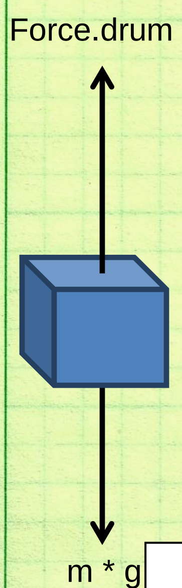

The velocity drops off linearly as the fastest we can decelerate the mass is equal to gravity. If we decelerate the mechanism more quickly, the mass will continue to move.

The equation for the curve is derived in part 9.

8. Continued

b. **Show your calculations for the accelerations, forces and torques you will use for this task and discuss why you made these choices**

There are no constant values for the accelerations, forces or torques that are being applied in the system. The following equations give their values.



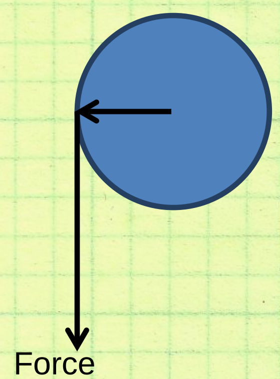

Free body diagram of weight

The net force on the weight is the sum of the force exerted by the drum and the force of gravity.

$$F_{net} = F_{drum} - mg$$

$F_{drum}$ falls in the following range (calculated using the equations in section 9)

$$mg \leq F_{drum} \leq 6.16\ N$$




Force acting on drum

The force exerted by the drum at a given torque can be found by dividing the torque of the drum by its radius.

$$F_{drum} = \frac{\tau_{drum}}{r_{drum}}$$

The torque of the drum is given by the following equation. This is derived from the equation for angular velocity of the drum in terms of torque. ω is angular velocity.

$$\tau_{drum} = \frac{-(\omega_{drum} - \omega_{drum.free})\tau_{drum.stall}}{\omega_{drum.free}} - (\tau_{gears})(R)$$

8. Continued

The $\tau_{gears}$ term comes from the torque needed to move the gear train. It is given by the following equation where $I_{eq}$ is the reflected inertia of the gear train as seen by the motor and $\alpha_{motor}$ is the angular acceleration of the motor.

$$\tau_{gears} = (I_{eq})(\alpha_{motor})$$

The torque will always satisfy the following inequality

$$\tau_{drum} \leq \tau_{drum.stall} = (\tau_{motor.stall})(R\,) = 4.771\ Nm$$

The acceleration of the weight is given by the following equation. It is derived in section 9.

$$a = \frac{\tau_{drum.stall}}{(m)(r_{drum})} - \frac{v_{weight}(\tau_{drum.stall})}{(m)(\omega_{drum.free})(r_{drum})^2} - \frac{(R)^2(I_{eq})(a)}{(m)(r_{drum})^2} - g$$

#9

**9.** Show and discuss your calculations for lift dynamics and total time for the lift.

Derivation of Position and Velocity Curves:

**Nomenclature**

$\tau$: Torque

$\omega$: Angular Velocity

$v$: Linear Velocity

$F$: Force

$I$: Inertia

$R$: Reduction

$r$: Radius

$m$: Mass

$a$: Linear Acceleration

$g$: Acceleration from Gravity (9.8 m/s)

$\alpha$: Angular Acceleration

9. Continued

Speed of Drum. This equation is derived from the linear change in the speed of the drum from free speed to stall.

$$\omega_{drum} = \frac{-\omega_{drum.free}}{\tau_{drum.stall}}\tau_{drum} + \omega_{drum.free} \qquad \text{eq. 1}$$

Solving eq. 1 for Torque at the drum.

The $\tau_{gears}$ term is subtracted because some of the power from the motor is used in accelerating the gear train:

$$\tau_{drum} = \frac{-(\omega_{drum} - \omega_{drum.free})\tau_{drum.stall}}{\omega_{drum.free}} - (\tau_{gears})(R) \qquad \text{eq. 2}$$

Force on the drum

$$F_{drum} = \frac{\tau_{drum}}{r_{drum}} \qquad \text{eq. 3}$$

Substituting eq. 2 into eq. 3

$$F_{drum} = \frac{\tau_{drum.stall}}{r_{drum}} - \frac{(\omega_{drum})(\tau_{drum.stall})}{(r_{drum})(\omega_{drum.free})} - \frac{(R)(\tau_{gears})}{r_{drum}} \qquad \text{eq. 4}$$

Net force on the weight (see free body diagram in part 8)

$$F_{Net} = F_{drum} - mg \qquad \text{eq. 5}$$

Acceleration in terms of force and mass

$$a = \frac{F_{Net}}{m} \qquad \text{eq. 6}$$

Substituting eq. 5 into eq. 6

$$a = \frac{F_{drum} - mg}{m} \qquad \text{eq. 7}$$

Substituting eq. 4 into eq. 7

$$a = \frac{\tau_{drum.stall}}{(m)(r_{drum})} - \frac{(\omega_{drum})(\tau_{drum.stall})}{(m)(r_{drum})(\omega_{drum.free})} - \frac{(R)(\tau_{gears})}{(m)(r_{drum})} - g \qquad \text{eq. 8}$$

9. Continued

Free speed of drum in terms of linear velocity of weight

$$\omega_{drum} = \frac{v_{weight}}{r_{drum}} \qquad \text{eq. 9}$$

Substituting eq. 9 into eq. 8

$$a = \frac{\tau_{drum.stall}}{(m)(r_{drum})} - \frac{v_{weight}(\tau_{drum.stall})}{(m)(\omega_{drum.free})(r_{drum})^2} - \frac{(R)(\tau_{gears})}{(m)(r_{drum})} - g \qquad \text{eq. 10}$$

Torque of the gears.

$I_{eq}$ is the inertia as “seen” by the motor. It is also known as reflected inertia.

$$\tau_{gears} = (I_{eq})(\alpha_{motor}) \qquad \text{eq. 11}$$

Angular acceleration of the motor in terms of acceleration of the drum

$$\alpha_{motor} = (\alpha_{drum})(R) \qquad \text{eq. 12}$$

Angular acceleration of the drum in terms of acceleration of the weight

$$\alpha_{drum} = \frac{a}{r_{drum}} \qquad \text{eq. 13}$$

Substituting eq. 13 into eq. 12

$$\alpha_{motor} = \frac{(R)(a)}{r_{drum}} \qquad \text{eq. 14}$$

Substituting eq. 14 into eq. 11

$$\tau_{gears} = \frac{(I_{eq})(R)(a)}{r_{drum}} \qquad \text{eq. 15}$$

9. Continued

Substituting eq. 15 into eq. 10

$$a = \frac{\tau_{drum.stall}}{(m)(r_{drum})} - \frac{v_{weight}(\tau_{drum.stall})}{(m)(\omega_{drum.free})(r_{drum})^2} - \frac{(R)^2(I_{eq})(a)}{(m)(r_{drum})^2} - g \qquad \text{eq. 16}$$

Rearranging terms

$$a + \frac{(R)^2(I_{eq})(a)}{(m)(r_{drum})^2} + \frac{v_{weight}(\tau_{drum.stall})}{(m)(\omega_{drum.free})(r_{drum})^2} - \left(\frac{\tau_{drum.stall}}{(m)(r_{drum})} - g\right) = 0 \qquad \text{eq. 17}$$

Let

$$A = 1 + \frac{(R)^2(I_{eq})}{(m)(r_{drum})^2} \qquad \text{eq. 18}$$

Let

$$B = \frac{\tau_{drum.stall}}{(m)(\omega_{drum.free})(r_{drum})^2} \qquad \text{eq. 19}$$

Let

$$D = -\frac{\tau_{drum.stall}}{(m)(r_{drum})} + g \qquad \text{eq. 20}$$

Rewriting eq. 11

$$Aa + Bv + D = 0 \qquad \text{eq. 21}$$

Rewriting as an Ordinary Differential Equation

$$A\frac{d^2x}{dt^2} + B\frac{dx}{dt} = -D \qquad \text{eq. 22}$$

Solving:

$$\int A\frac{d^2x}{dt^2} + B\frac{dx}{dt}\,dt = \int -D\,dt \qquad \text{eq. 23}$$

$$A\frac{dx}{dt} + Bx = -Dt \qquad \text{eq. 24}$$

9. Continued

$$\left(e^{\frac{B}{A}t}\right)\left(\frac{dx}{dt}+\frac{B}{A}x\right)=(e^{\frac{B}{A}t})(-\frac{D}{A}t) \qquad \text{eq. 25}$$

$$\frac{d}{dt}\left(xe^{\frac{B}{A}t}\right)=(e^{\frac{B}{A}t})(-\frac{D}{A}t) \qquad \text{eq. 26}$$

$$xe^{\frac{B}{A}t}=\int(e^{\frac{B}{A}t})(-\frac{D}{A}t) \qquad \text{eq. 27}$$

$$xe^{\frac{B}{A}t}=\left(-\frac{D}{A}\right)\left(\frac{te^{\frac{B}{A}t}}{\frac{B}{A}}-\frac{e^{\frac{B}{A}t}}{\left(\frac{B}{A}\right)^2}\right)+C \qquad \text{eq. 28}$$

$$xe^{\frac{B}{A}t}=\left(\frac{-D}{B}\right)\left(te^{\frac{B}{A}t}-\frac{e^{\frac{B}{A}t}}{\frac{B}{A}}\right)+C \qquad \text{eq. 29}$$

$$x=\left(\frac{-D}{B}\right)\left(t-\frac{A}{B}\right)+Ce^{-\frac{B}{A}t} \qquad \text{eq. 30}$$

$$x(0)=0 \qquad \text{eq. 31}$$

$$0=\left(\frac{-D}{B}\right)\left(0-\frac{A}{B}\right)+Ce^{0} \qquad \text{eq. 32}$$

$$0=\left(\frac{-D}{B}\right)\left(\frac{-A}{B}\right)+C \qquad \text{eq. 33}$$

$$C=-\frac{AD}{B^2} \qquad \text{eq. 34}$$

$$x=\left(\frac{-D}{B}\right)\left(t-\frac{A}{B}\right)-\left(\frac{AD}{B^2}\right)e^{-\frac{B}{A}t} \qquad \text{eq. 35}$$

Solution

$$x=\left(\frac{-AD}{B^2}\right)\left(e^{-\frac{B}{A}t}-1\right)-\left(\frac{D}{B}\right)(t) \qquad \text{eq. 36}$$

9. Continued

The velocity curve (Section 8.a.) is derived by differentiating eq. 36 with respect to t.

$$v = \left(\frac{D}{B}\right)\left(e^{-\frac{B}{A}t} - 1\right) \qquad \text{eq. 37}$$

Solution for Time for Lift :

$$x = 2 \qquad \text{eq. 38}$$

Total time:

$$t \cong 1.11 \qquad \text{eq. 39}$$

Deceleration Time:

Since continuing to drive the weight at maximum velocity would damage the mechanism, the weight must begin decelerating before it reaches the target height. Optimally, this deceleration would begin as late as possible to minimize the lift time. Since the weight is supported on a string, the maximum value of deceleration is 9.8 m/s, as the weight would continue independent of the motor. From a practical standpoint, the motor must be isolated from the load when it is stopped, or the power must be removed and the mass falls to prevent damage to the motor.

9. Continued

**Derivation of Deceleration time:**

The time required to reach a given velocity is given by the following equation

($v_{final}$ = 0)

$$t = \frac{v_{final} - v_0}{a} = \frac{-v_0}{-g} \qquad \text{eq. 40}$$

Substituting this value of t into the equation for projectile motion (our weight will optimally be decelerating as close to the maximum deceleration (gravity) as possible.

$$x = \frac{-gt^2}{2} + v_0 t + x_0 = -\frac{v_0^2}{2g} + \frac{v_0^2}{g} + x_0 \qquad \text{eq. 41}$$

Replacing x with 2 (our target height) and substituting eq. 36 and eq. 37 for $x_0$ and $v_0$ respectively.

$$0 = \left(\frac{D^2}{B^2}\right)\left(e^{\frac{2B}{A}t} - 2e^{-\frac{B}{A}t} + 1\right)\left(\frac{1}{2g^2} + \frac{1}{g}\right) + \left(-\frac{AD}{B^2}\right)\left(e^{-\frac{B}{A}t} - 1\right) - \left(\frac{D}{B}\right)(t) - 2$$

eq. 42

Solution for optimal time (seconds) at which to begin deceleration:

$$t \cong 1 \qquad \text{eq. 43}$$

The position and velocity after t = 1 are given by the equations:

$$x = -\frac{g}{2}t^2 + v(1)t + x(1) \qquad \text{eq. 44}$$

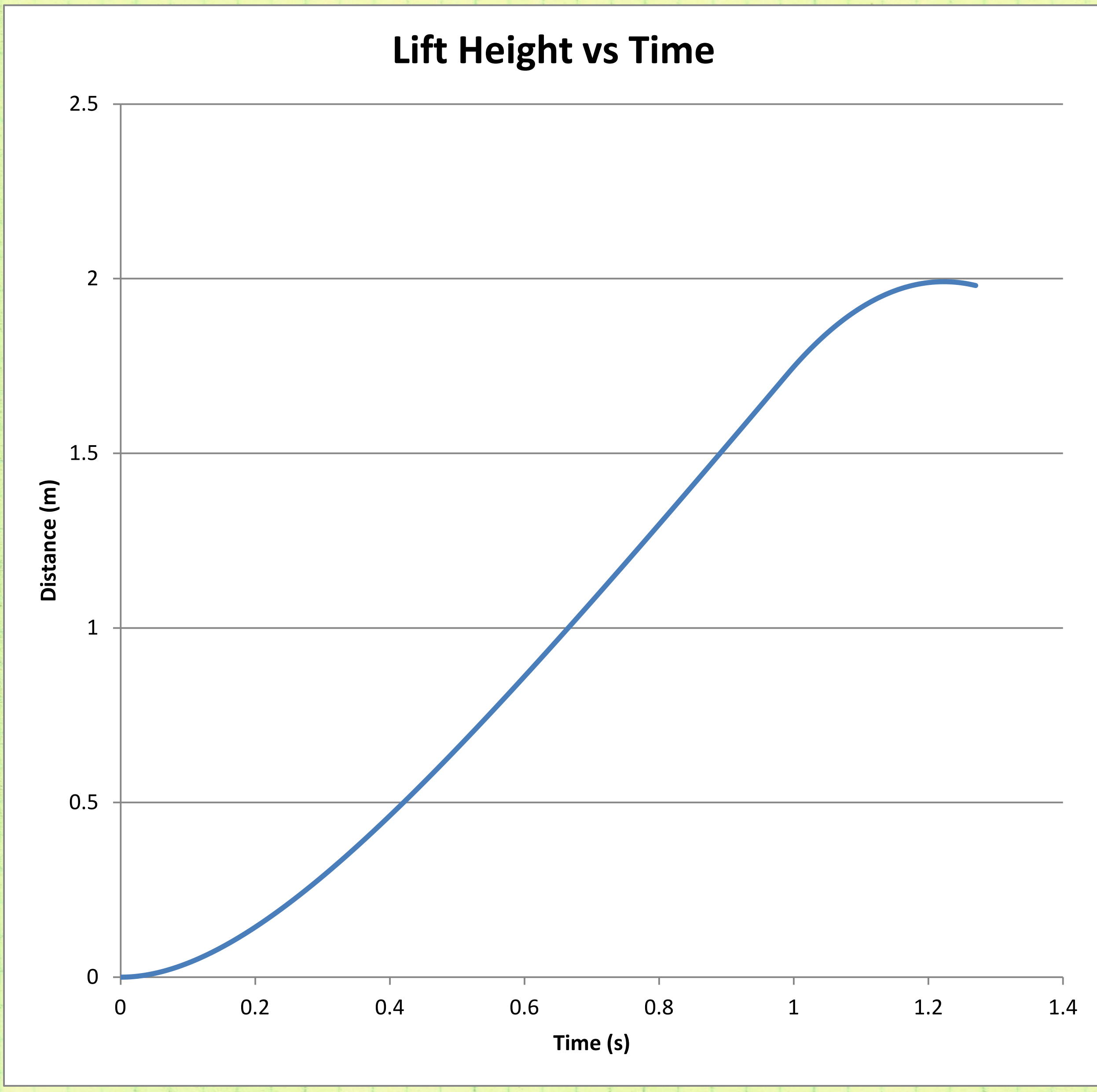
$$v = -gt + v(1) \qquad \text{eq. 45}$$

Based on these, the total time for the lift (with optimal deceleration) in seconds is:

$$t \cong 1.23 \qquad \text{eq. 46}$$

9. Continued

Plot of lift position (using eq. 36 and eq. 44):

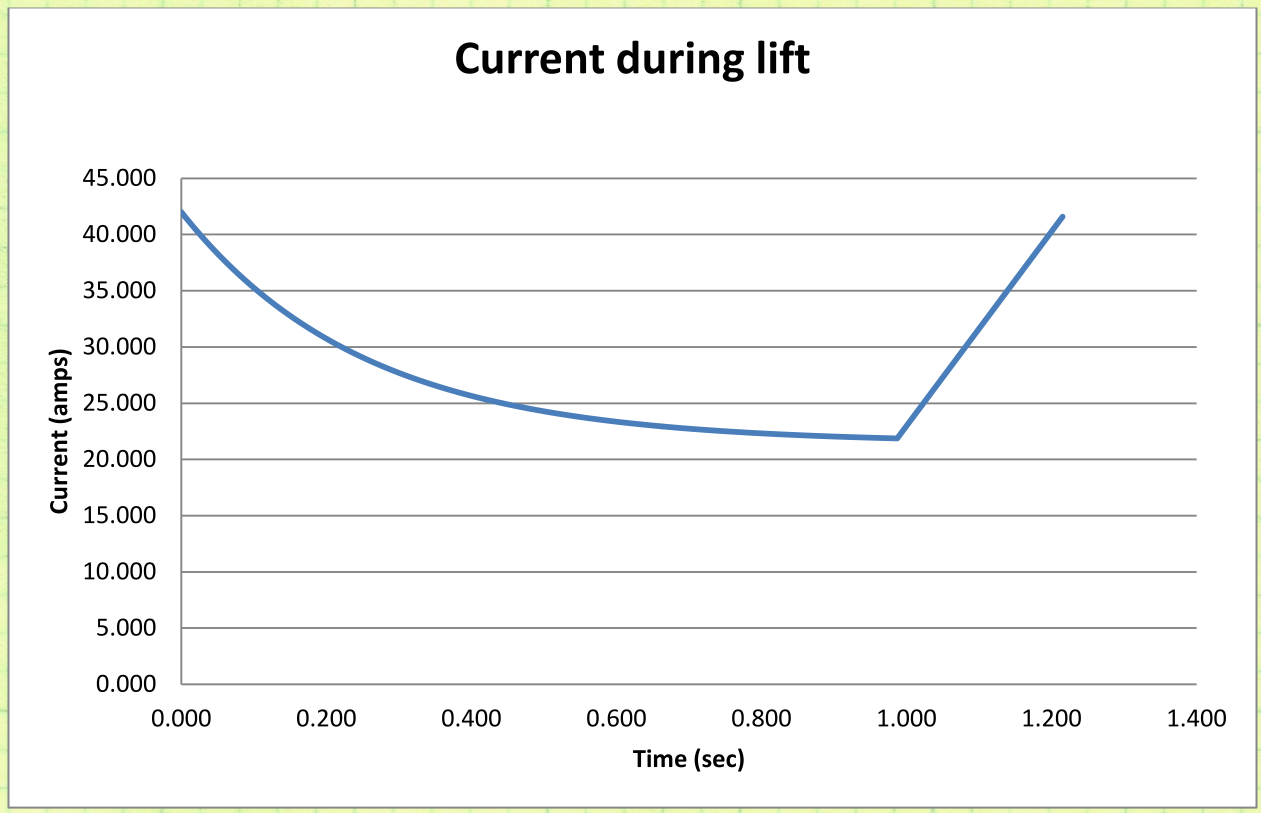


# #10

10. Show and discuss your calculations for how many times this lift can be repeated with a fully charged battery (assuming the batteries voltage stays at a constant voltage).

The battery we are using is a 12 volt 18 amp-hour battery. This means that the capacity of the battery is 18 amps for one hour (amp-hr) assuming that the voltage stays the same. So the battery capacity is 64,800 amp seconds (amp-sec). This number is equal to 18 amp-hour*60 sec/min*60 min/hour.

Each time the load is lifted the energy consumed from the battery (A) is 33.5 amp-sec. The graph below shows the current draw as a function of time throughout the lift.



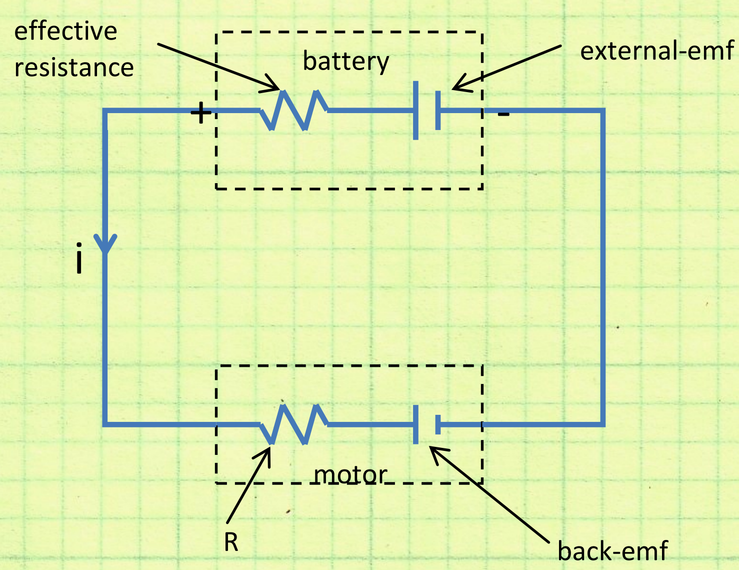

The energy consumed during each lift is the integral of this function between the start and stop. This is equal to the area under the curve. We calculated the value incrementally using Excel to find the total amount of energy used per lift. The number of lifts available is equal to the capacity of the battery divided by the energy consumed during each lift as shown in the equation below.

10.Continued

$$N_{lift} = \frac{capacity_{battery}}{energy_{lift}} = \frac{64{,}800\,amp - \sec s}{33.5\,\frac{amp - \sec s}{lift}} = 1934\,lifts$$

In reality we will not get this many lifts because the battery will not have as much capacity as the voltage declines over time.

The battery has an internal resistance that reduces the output voltage. As the battery ages or receives damage, this resistance increases, reducing the power supplied to the motor. When you apply a volt-meter to the leads of a battery, you may read 12V.  If the battery has a large internal resistance, the voltage could drop dramatically when a large current is drawn from the battery.




We would need to take both of these factors into account in order to get an accurate number of lifts.

$Power_{Loss} = i^2 \cdot R$   Power loss is described by this equation.

# #11

11. Using physics and math show how to derive the torque vs. speed motor curve.

The inversely proportional relationship observed between the motor torque and rotational speed is a product of reverse EMF. When the motor is functioning, current is moving through the motor's coils (which are in a magnetic field) causing a force on the armature that is the cross product of the field and the current flow. When the motor's armature moves in response to this force, the coil wires cross field lines, inducing an electromotive force that is directly opposite the current flowing through the motor (Faraday's Law of Induction, Lenz's Law). Back EMF in the motor can be expressed as:

$$EMF = BLv$$

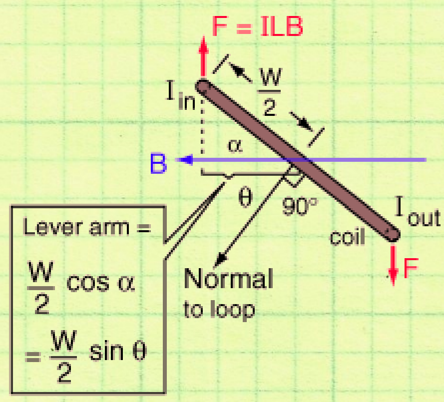



Image courtesy of Hyperphysics

B-Magnetic field (T), L=Length of wire (m), v= velocity of wire (m/s)

11.Continued

As the "$v$" (motor speed) term increases, so does the magnitude of the back EMF. Eventually, the back EMF and the motor input voltage draw equal. At this point, the motor's torque is zero and free speed, $\omega$, is at its maximum (conclusion 4). Likewise, at $\omega = 0$, the EMF is 0, meaning torque is at its peak. The generator constant (a value that is different for each motor), gives the slope between these two points by expressing how much torque is lost with increased velocity.  In general, the higher the number of turns on each coil the motor has, the larger this number is.

<u>Equations</u>

| | | |
|---|---|---|
| Ohm's law: (Eq.1) | $V = iR$ | |
| Law of Induction: | $EMF = N\frac{\Delta\Phi}{\Delta t}$ | |
| Lenz's Law: | $EMF = -N\frac{\Delta\Phi}{\Lambda t}$ | (Eq.2) |
| Lorentz Law: | $EMF = BLv$ | (Eq. 3) |
| Power | $W = iV$ | (Eq.4) |
| Conservation of energy dictates: | $P_{in} = P_{mech} + P_{loss}$ | (Eq.5) |
| Substitution into Eq.5 | $iV = \tau\omega + i^2R$ | (Eq.5B) |
| Ohm's Law/Faraday's law: | $V_{motor} = (iR + EMF)$ | (Eq.6) |
| Definition of velocity | $v = \omega r$ | (Eq.7) |

11. Continued

Substitution into Eq.3 $EMF = \omega(BLr)$ (Eq.8)

*(B,L and r are not available)*

Rename BLr "$K_n$" $EMF = K_n\omega$ (Eq.10)

Conclusion 1: EMF ∝ speed

Plug in $Eq$. 10 to Eq. 6 $V_{motor} = iR + K_n(\omega)$ (Eq. 11)

$V_{motor=}$-EMF-iR=0 (Eq.11B)

Set equal to Eq. 5B $W = i^2R + i(K_n)(\omega)$ (Eq. 12)

$$\tau\omega + i^2R = i^2R + i(K_n)(\omega)$$

Cancel out $\tau = K_{motor}(i)$ (Eq.12)

Conclusion 2: Torque ∝ current

Rewrite Eq.12 $i = \frac{\tau}{K_{motor}}$ (Eq.13)

Substitute Eq.10, Eq.13 into Eq.11B $V_{motor}\frac{\omega}{K_n} - \frac{\tau}{K_{motor}}(R)$ (Eq.14)

11. Continued

Manipulating Eq. 14 provides the answer of:

$$\omega = K_n(V_{motor} - \frac{\tau}{K_{motor}} * R) \quad \text{(Eq.15)}$$

where:

Ohm's law $$R = \frac{V_{motor}}{i_{stall}} \quad \text{( Eq.16)}$$

By manipulating equation 15, one can derive:

$$\tau_{stall} = V_{motor}\left(\frac{K_m}{R}\right)$$ torque is highest at $\omega = 0$

$$i_{stall} = \frac{\tau_{stall}}{K_m}$$ From Eq. 13

$$\omega_{free} = K_n(V_{motor})$$ Speed is max when $\tau = 0$

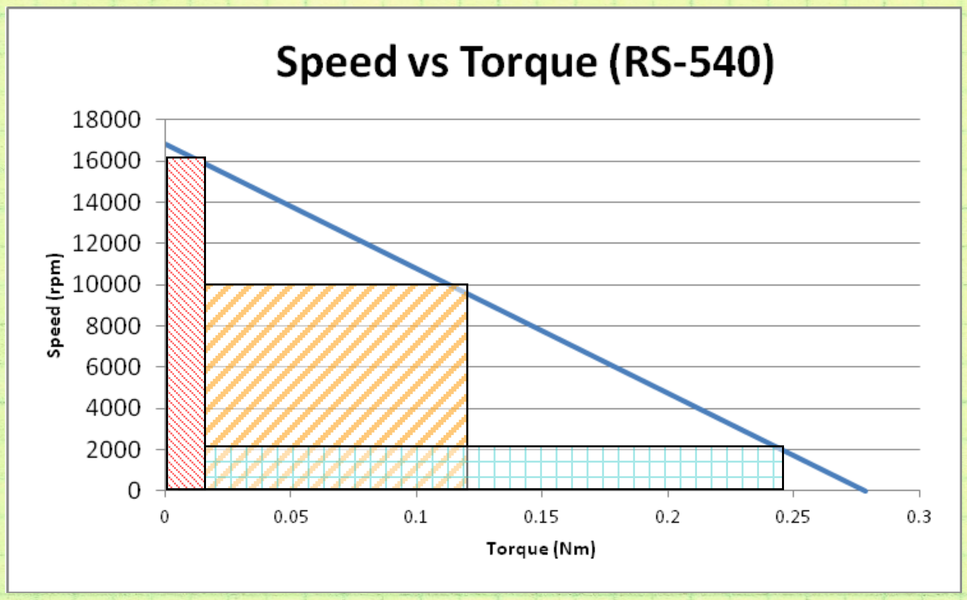
$K_n$ *and* $K_m$ are constants available from the motor manufacturer.

This set of equations is significant because it presents the necessary equations to develop the slope and intercepts of the line that denotes the relationship between motor speed and torque. The first conclusion, Eq. 10, states that the reverse electromotive force increases with motor speed. As shown in part three, the increase in back EMF corresponds to a decrease in torque. The slope of this relationship is given by the term $K_N$. This line's intercepts on the $\tau$ and $\omega$ axes are dependent on the motor input voltage. This means that a higher input voltage increases both stall torque and free speed, without changing the slope between the two points (aka moving up and down Y axis). In our challenge, this data is valuable in that it assists us in determining the torque available at a given speed and voltage, which in turn allows us to determine reduction ratios and lift times.

#12

12. Using physics and math show how to derive the power curve.

The power output of the motor is given by the area of a rectangle which has one corner anchored to the line and the other three on the axes (see image below). As you move to the right, the $\omega$ term increases as the $\tau$ term decreased, eventually reaching zero at $\omega = max$. This relationship (albeit in reverse) holds true moving left. The max power occurs at ½ torque and ½  , decreasing on either side. Using either substitution or by picking points and rectangle areas, it can be proven that power is modeled with an inverted parabola with intercepts at $(\omega = 0, \tau = \text{max}) and\ (\omega = max, \tau = 0)$.




<u>Math:</u>

The object of this set of calculations is to demonstrate that power is equal to torque*angular velocity.

| | | |
|---|---|---|
| Power is by definition: | $P = \frac{\Delta w}{\Delta t} = \Delta \text{f} * \frac{\Delta \text{x}}{\Delta \text{t}}$ | Eq.1a/b |
| | $w = f * d$ | Eq.2 |
| | $v = \omega * r$ | Eq.3 |
| | $\tau = f * r$ | Eq.4 |
| Rewriting Eq.3 | $r = \frac{v}{\omega}$ | Eq.5 |

12. Continued

| | | |
|---|---|---|
| Substitution of Eq.5 into Eq.4 | $\tau = f * \frac{\nu}{\omega}$ | Eq.6 |
| Manipulate | $\tau * \omega = f * \nu$ | Eq.7 |
| From Eq.3/definition of velocity | $\nu = \frac{\Delta x}{\Delta t}$ | Eq.8 |
| Part 2 of Eq.9 matches Eq.1b | $\tau * \omega = f * \frac{\Delta x}{\Delta t}$ | Eq.9 |

Therefore,

$$Power = \tau * \omega$$